**Goal**

* Estimate vector to rocket in body frame
* Estimate gyro biases

**Two models to observe gyro biases**

* A single vector observation leaves ambiguity about gyro biases, need two to make them fully observable
* Magnetic field gives observability to gyro biases in two ways:

1. Use measured rate of change of magnetic field to observe gyro bias
   * In the body frame:
   * Can create measurement model for based on measured gyro, estimated bias, and predicted:
   * Residual becomes difference in measured (from finite difference) and expected from measured gyro and estimated bias (above)
   * Relies on finite differences, which will cause much more noise in the measurement
   * Might? Become messier in the observation model (I haven’t worked out the math yet)
2. Track body magnetic field as another state variable and let the STM do the work for us
   * We are already tracking another inertial vector in the body frame: the direction to the rocket, this makes our bias observable through the STM, adding B to the estimate improves that
   * Estimate next based on measured gyro and estimated bias 🡪 measure next (measuring a state variable directly so becomes the identity matrix 🡪 residual tells us how much to correct our bias based on inaccurately propagated state
   * Does not rely on noisy finite difference, is not truly measurable in a nice way
   * **This is clearly the superior option (until proven otherwise)**

**Measurements**

* Magnetometer 🡪 senses body frame geomagnetic field
* Gyroscope 🡪 senses body angular rates
* Rocket direction sensor (IR/optical/RF) restricted to a certain FOV 🡪 senses bod frame direction to rocket
  + This should really be an EKF based on the raw observables (intensity, phase, pixel location, etc.) but we haven’t designed the sensor yet so here we are

**State vector**

* : body frame vector to rocket
* : body frame magnetic field
* : gyroscope bias

**State Dynamics**

Assuming no change in inertial magnetic field (which is appropriate for small translational motion) and no change in inertial vector to the rocket (appropriate for small relative motion between the vehicles – IDK how appropriate that assumption is), dynamics of the target vector and magnetic field are:

With dynamics model replacement (using measured gyro and estimated bias) we get:

For estimate purposes we assume the bias does not change:

Putting these together we find:

We can write this in state-space matrix form as:

The state transition matrix can be found via:

**Measurement Model**

We directly observe the target vector and the magnetic field, so the observation model is simply:

**Kalman Filter Algorithm**

Initialize (do this once)

Set initial a-priori covariance

Set initial state estimate

Set process and measurement noise matrices and

Update

Compute Kalman gain

Take measurements:

Update state to a-priori

Update estimate covariance

Predict

**Now realizing the update step will not obey the norm constraint on the target vector (or normalized b-field if that is used) which could be a problem 🡪 why people do multiplicative extended filters. I think it should e fine though for small updates and brute-forcing the renormalization (maybe). With low uncertainty in the measurement of the target vector the estimate will essentially always bounce back to what was measured so it might be fine.**